

Thermal Abelian monopoles as selfdual dyons.

V. G. Bornyakov

*High Energy Physics Institute, 142280 Protvino, Russia
and Institute of Theoretical and Experimental Physics, 117259 Moscow, Russia*

V. V. Braguta

High Energy Physics Institute, 142280 Protvino, Russia

The properties of the thermal Abelian monopoles are studied in the deconfinement phase of the $SU(2)$ gluodynamics. To remove effects of Gribov copies the simulated annealing algorithm is applied to fix the maximally Abelian gauge. To study monopole profile we complete the first computations of excess of the nonabelian action density as a function of the distance from the center of the thermal Abelian monopole. We have found that starting from the distances ≈ 2 lattice spacings the chromoelectric and chromomagnetic action densities created by monopole are equal to each other, from what we draw a conclusion that monopole is a dyon. Furthermore, we find that the chromoelectric and chromomagnetic fields decrease exponentially with increasing distance. These findings were confirmed for different temperatures in the range $T/T_c \in (1.5, 4.8)$.

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One of the hypotheses which have been put forward in the recent past is that the quark-gluon plasma (QGP) properties might be dominated by a magnetic component [1–3]. The monopoles or center vortices might be responsible for unexpected properties of the hadron matter at $T > T_c$: on one hand it is well known from lattice results that the equation of state is close to that of an ideal gas, on the other hand the very low viscosity to entropy ratio tells that it is an ideal liquid.

In Ref. [2] such magnetic component has been related to thermal Abelian monopoles evaporating from the magnetic condensate which is believed to induce color confinement at low temperatures. Moreover it has been proposed to detect such thermal monopoles in finite temperature lattice QCD simulations, by identifying them with monopole currents having a non-trivial wrapping in the Euclidean temporal direction [2, 4, 5].

The way one can study the monopoles properties on the lattice is via an Abelian projection after fixing the maximally Abelian gauge (MAG) [6–8]. This gauge as well as the properties of the monopole clusters has been investigated in a numerous papers both at zero and nonzero temperature (see for extensive list of references, e.g. [9]). The evidence was found that the nonperturbative properties of the gluodynamics such as confinement, deconfining transition, chiral symmetry breaking, etc. are closely related to the Abelian monopoles defined in MAG. This was called a monopole dominance.

First numerical investigations of the wrapping monopole trajectories were performed long ago in Refs. [4] and [5]. A more systematic study of the thermal monopoles in $SU(2)$ Yang-Mills theory at high temperature has been performed in Refs. [10–12]. In

particular, it was found in [10] that the density of monopoles is independent of the lattice spacing, as it should be for a physical quantity.

In paper [11] very interesting properties of the thermal Abelian monopoles were found. The authors measured the excess of chromoelectric and chromomagnetic action density on the surface of the lattice (hyper-)cubes with monopoles inside. The dependence of the excess on the distance from the monopole center was determined through the variation of the lattice spacing a (similar investigation at $T = 0$ was made in [13]). As a result it was found that with good accuracy the chromomagnetic and chromoelectric action densities created by monopole have the following behavior: $H^2(r), E^2(r) = a_{H,E}/r^4$. The coefficients a_H and a_E turned out to be equal to each other with a very good accuracy, from what the authors concluded that monopole is a dyon. It is worth to note that there were other works in the past where dyonic properties of the monopoles were observed [14–16].

The drawback of the study undertaken in [11] is that all results were obtained at the ultraviolet cutoff scale and were thus subjected to both lattice discretization errors and ultraviolet divergences. In view of the importance of the findings of [11] in this paper we are going to study the chromoelectric and chromomagnetic fields created by monopole and to check whether the observed behavior is correct or is just a lattice artifact, i.e. artifact of the ultraviolet cut off. To accomplish this check we will measure the chromoelectric and chromomagnetic fields at various distances from the monopole center.

In this paper we study the $SU(2)$ lattice gauge theory with the standard Wilson action

$$S = \beta \sum_x \sum_{\mu>\nu} \left[1 - \frac{1}{2} \text{Tr} \left(U_{x\mu} U_{x+\mu;\nu} U_{x+\nu;\mu}^\dagger U_{x\nu}^\dagger \right) \right],$$

where $\beta = 4/g_0^2$ and g_0 is a bare coupling constant. The link variables $U_{x\mu} \in SU(2)$ transform under gauge transformations g_x as follows:

$$U_{x\mu} \xrightarrow{g} U_{x\mu}^g = g_x^\dagger U_{x\mu} g_{x+\mu}; \quad g_x \in SU(2). \quad (1)$$

Our calculations were performed on the asymmetric lattices with lattice volume $V = L_t L_s^3$, where $L_{t,s}$ is the number of sites in the time (space) direction. The temperature T is given by

$$T = \frac{1}{aL_t}, \quad (2)$$

where a is the lattice spacing.

The MAG is fixed by finding an extremum of the gauge functional

$$F_U(g) = \frac{1}{4V} \sum_{x\mu} \frac{1}{2} \text{Tr} \left(U_{x\mu}^g \sigma_3 U_{x\mu}^{g\dagger} \sigma_3 \right), \quad (3)$$

with respect to gauge transformations g_x . We apply the simulated annealing (SA) algorithm which proved to be very efficient for this gauge [17] as well as for other gauges such as center gauges [18] and Landau gauge [19]. To further decrease the Gribov copy effects we generated 10 Gribov copies starting every time gauge fixing procedure from a randomly selected gauge copy of the original Monte Carlo configuration.

In Table I we provide the information about the gauge field ensembles used in our study.

β	$a[\text{fm}]$	L_t	L_s	T/T_c	N_{meas}
2.43	0.108	4	32	1.5	1000
2.5115	0.081	4	28	2.0	400
2.635	0.054	4	36	3.0	500
2.80	0.034	4	48	4.8	400

TABLE I: Values of β , lattice sizes, temperatures, number of measurements and number of gauge copies used throughout this paper. To fix the scale we take $\sqrt{\sigma} = 440$ MeV.

The chromomagnetic action density at a site x is defined as

$$S_M(x) = \frac{1}{12} \sum_{P_s \ni x} \left(1 - \frac{1}{2} \text{Tr} U_{P_s} \right). \quad (4)$$

The sum is taken over all spatial plaquettes P_s which contain the lattice site x . In the continuum limit this

expression is proportional to $\sim \text{Tr}(G_{23}^2 + G_{13}^2 + G_{12}^2) = \text{Tr}(H_1^2 + H_2^2 + H_3^2)$. So, this expression can be taken as a measure of the chromomagnetic action $\text{Tr}(\mathbf{H}^2)$ at the site x .

Analogously, for the chromoelectric action density at a site x we take

$$S_E(x) = \frac{1}{12} \sum_{P_t \ni x} \left(1 - \frac{1}{2} \text{Tr} U_{P_t} \right). \quad (5)$$

Here the sum is taken over all time-like plaquettes P_t which contain the site x . In the continuum limit this expression is proportional to $\sim \text{Tr}(G_{01}^2 + G_{02}^2 + G_{03}^2) = \text{Tr}(E_1^2 + E_2^2 + E_3^2)$ and thus it can be taken as a measure of the chromoelectric action $\text{Tr}(\mathbf{E}^2)$. Note that our definitions for $S_{M,E}(x)$ differ from those used in Ref. [11]. Although definitions of [11] were natural for the surface of a cube with monopole our definitions are more suitable for measurements at some distance from such cube.

Since we are studying the fields created by a monopole we should subtract the vacuum fluctuations of the chromomagnetic and chromoelectric actions from the equations (4), (5). We define the excess of the action density as

$$\langle \delta S_{M,E}(d) \rangle = \overline{\langle S_{M,E}(x) \rangle} - \langle S_{M,E} \rangle, \quad (6)$$

where $\langle \dots \rangle$ means ensemble average, d is the distance from a monopole and bar means averaging over all wrapped monopoles and all sites x at the distance d from monopole centers.

The monopole currents and their wrapping numbers are defined in a standard way (see e.g. [10]). Moving along wrapped monopole clusters on a dual lattice we detect all 3-dimensional cubes in all time slices on the original lattice which contain monopoles corresponding to j_4 component of the magnetic current. Having detected all such 3-dimensional cubes we do the measurements of the chromomagnetic and chromoelectric action densities $\langle \delta S_{M,E}(d) \rangle$ at various distances from a given monopole and then we take the average over all thermal monopoles found on the lattice.

Now let us consider a three dimensional cube with the monopole belonging to a wrapped cluster. Below it will be assumed that the monopole is located in the center of this cube and we take the center as a coordinate origin. Actually, one cannot assert that the monopole is exactly located at the center of the cube. However, since we take an average over all monopoles, this approximation can be considered as a good one. We have measured the action densities $\langle \delta S_{M,E}(d) \rangle$ at various distances d from the centers of the cubes with monopole. The distance was defined as a length of the vector $\vec{d} = \{n_1 \pm 1/2, n_2 \pm 1/2, n_3 \pm 1/2\}$ from the coordinate origin to lattice site x under consideration. We present results of measurements for $\vec{d} = \frac{1}{2}\{m, m, m\}$, $m = 1, 3$ and $\vec{d} = \frac{1}{2}\{1, 1, 3\}$,

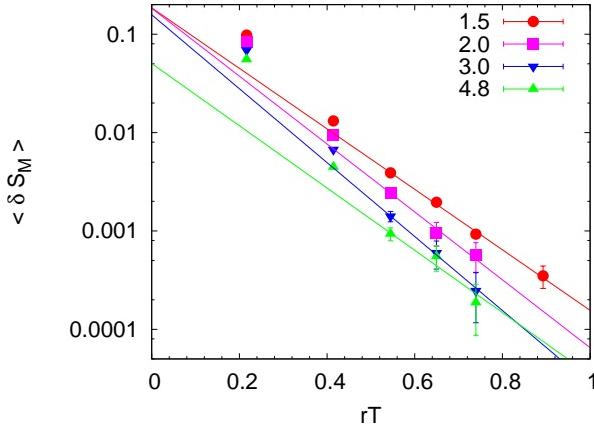


FIG. 1: $\langle \delta S_M \rangle$ defined in eq.(6) as function of the dimensionless distance rT from the monopole center at the temperatures $T/T_c = 1.5, 2.0, 3.0, 4.8$.

$\frac{1}{2}\{1, 3, 3\}$, $\frac{1}{2}\{1, 1, 5\}$, $\frac{1}{2}\{1, 3, 5\}$. For $T/T_c = 1.5$ additionally results for $\vec{d} = \frac{1}{2}\{1, 5, 5\}$, $\frac{1}{2}\{1, 1, 7\}$ are presented. For longer distances the statistical errors were too large.

In Figures 1 and 2 results are shown for the $\langle \delta S_M \rangle$ and $\langle \delta S_E \rangle$ as functions of the dimensionless distance $rT = d/4$. From these Figures one clearly sees that at least at large distances both $\langle \delta S_M \rangle$ and $\langle \delta S_E \rangle$ decrease with distance in agreement with exponential fall-off $\sim \exp(-2M_{m,e}r)$. The dependence $1/r^4$ found in [11] is ruled out. We do not have enough data points to determine the pre-exponential function by fitting. Respectively, it is rather difficult to find the parameters $M_{m,e}$ with a good accuracy. In this paper we just make rough estimation of these parameters fitting the last 3 data points for $\langle \delta S_M \rangle$ to the exponential fall-off with constant prefactor. We get the following results: $M_m/T = 3.5(2), 4.0(4), 4.3(2), 3.7(8)$ for the temperatures $T/T_c = 1.5, 2.0, 3.0, 4.8$, respectively.

Looking at Figures 1, 2 one can see that the data lie on the smooth curves. This means that the data obey rotational invariance, since the data at different distances were measured in different directions. Moreover, vectors $\vec{d} = \frac{1}{2}\{3, 3, 3\}$ and $\vec{d} = \frac{1}{2}\{1, 1, 5\}$ have equal length and one can check the rotational invariance directly. Indeed, we find for these two vectors consistent results with deviations within 2σ interval. In all Figures we show averaged data for these vectors \vec{d} .

At large enough distances (beginning from the distance $d = 2.18$) the monopole chromomagnetic and chromoelectric action density seem to be equal to each other. To demonstrate this important property we plot the ratio $\langle \delta S_M \rangle / \langle \delta S_E \rangle = H^2/E^2$ in Figure 3. From this Figure we see that within the error bars at distances $rT \gtrsim 0.5$ the ratio $\langle \delta S_M \rangle / \langle \delta S_E \rangle$ is compatible with 1. From this observation one can

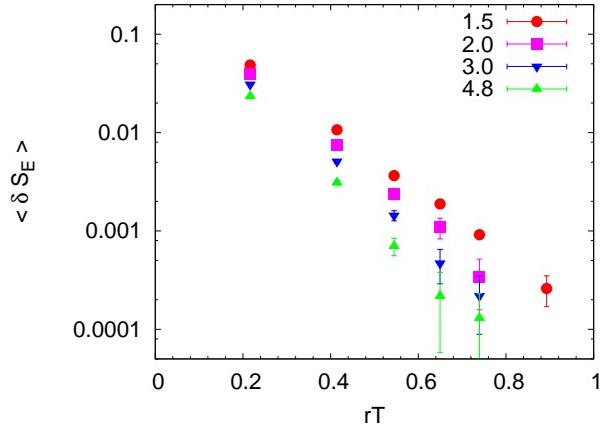


FIG. 2: Same as in Figure 1 for $\langle \delta S_E \rangle$.

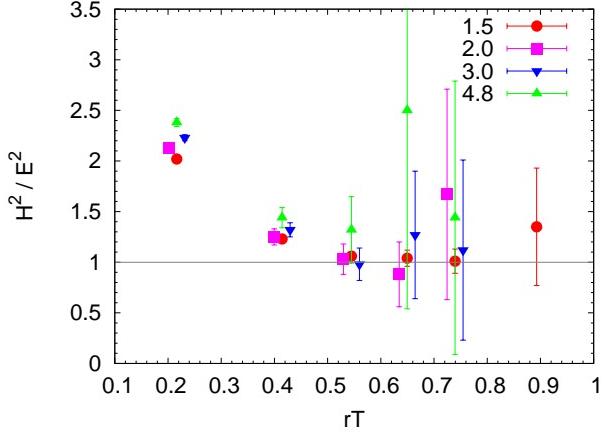


FIG. 3: The ratio $\langle \delta S_M \rangle / \langle \delta S_E \rangle = H^2/E^2$ as function of the distance rT from the monopole center at temperatures $T/T_c = 1.5, 2.0, 3.0, 4.8$. The data sets at $T/T_c = 2.0$ and 3.0 are shifted along horizontal axis to improve readability of the figure.

draw a conclusion: at least at large enough distances $H^2(r) = E^2(r)$. This implies that *monopoles carry both chromoelectric and chromomagnetic charges and they are equal*. So *monopoles are selfdual dyons*. These statements are the main results of this paper.

To understand the reason of lack of selfduality at small distances let us look at Figure 3. For all temperatures we observe similar behavior. At the distance $d = 0.87$ in lattice units the ratio $\langle \delta S_M \rangle / \langle \delta S_E \rangle \sim 2$ for all temperatures. At the distance $d = 1.66$ the ratio $\langle \delta S_M \rangle / \langle \delta S_E \rangle \sim 1.3$. At larger distances the ratio is compatible with unity. We believe that deviation of the ratio from one at small distances can be explained by the discretization effects. Notice that our definition of the chromomagnetic and chromoelectric action densities at site x is nonlocal involving all plaquettes in respective planes which own site x . This nonlocality is different for two action densities: it is purely

spatial for the chromomagnetic action density and is both spatial and temporal for the chromoelectric one. Thus at distances of order of one lattice spacing we, evidently, measure the fields taken at different points and different distances. These our arguments should be checked by computations with smaller lattice spacing, i.e. with $L_t > 4$. This will be done in a forthcoming paper. In that paper we will also present our data on the density and interactions of the thermal Abelian monopoles.

It is clear that results and conclusions of Ref. [11] where $\langle \delta S_M \rangle$ and $\langle \delta S_E \rangle$ were measured at the nearest possible distance to the monopole center are subjected to same discretization effects as discussed above for our data at small distances. While the distance dependence $1/r^4$ found in Ref. [11] is an ultraviolet divergence effect.

Thus we established that the Abelian thermal monopoles carry both chromoelectric and chromo-

magnetic charges and they are equal. So monopoles are selfdual dyons. Furthermore respective action densities are screened. We believe that these results are important for understanding QCD in the quark-gluon plasma phase since many recent theoretical models of this phase include monopoles as an important ingredient. These are model of Ref. [1, 3] based on competition between magnetic and electric quasiparticles, dyon [20] and caloron [21] models.

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